CHAPTER 2

COMPLEMENTS ARITHMETICS AND BINARY CODES

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CME 221 LOGIC CIRCUITS

This Chapter Includes:

- **Complement Arithmetics**
- **r Complement Arithmetics**
- **r-1 Complement Arithmetics**
- **Binary Codes**
- **BCD Code**
- **Weighted Codes**
- **Non Weighted Codes**

COMPLEMENT ARITHMETICS

- **Complement arithmetics is for representing negative (-) numbers. For positive (+) numbers complement arithmetics is NOT used.**
- **In general for a radix (base) r (r can be 2, 10 or any other base) system there are two types of complements:**
- **a) r's complement**
- **b) r-1's comlement**
- **In binary number system 2's complement (r's complement) or 1's complenent (r-1 complement) is used**
- **In decimal systems 10' complement (r's complement) or 9's complement (r-1 complement) is used.**

r-Complement of negative number

In a radix (base) r system complement r of a negative number N , when n digits are used, is represented as:

$N_r = r^n - N$ (n: number of digits used.)

Example 1: Find the 10's complement of $(125.456)_{10}$

Number of digits in the integer part of $(-125.456)_{10}$ is 3.

Therefore $r^n = 10^3$. $r^n - N = 10^3 - 125.456 = 874.544$

Hence 10's complement of $(-125.456)_{10}$ is 874.544

EXAMPLE 2: Find 2's complements of $(-110010.1011)_2$.

In the iteger part of number $(110010.1011)_2$ there are 6 digits. Therefore $r^n = 2^6$, and
$r^n - N = 2^6 - 110010.1011 = 1000000 - 110010.1011 = 001101.0101$

In binary system 2's complement of a number can be obtained in two ways:

a) **Complement every bit of number N and add 1 to LSB**

Example 3: Find 2's complenent of 101101
Solution: First complenet every bit: 010010
Add 1 to LSB

010010

1

010011 (asnwer)

b) Starting from LSB repeat all zeros until firts 1 is obtaned. Keep firts 1 same and continue complementing other bits.

Example 4: Find the 2's complement of 1001100

Solution: Aplying the above rule answer will be:

0110100

EXAMPLE 5: Find the 10's complement of $(47.83)_{10}$ (r=10, n=2)

Solution: $N^r = 10^2 - 47.83 = 52.10$ olur.

EXAMPLE 6: Find the 2's complement of $(0101101.101)_2$.

Solution:

$N^r = 2^n - N = 10000000 - 0101101.101 =$

$= (1010010.011)_2$.

r-1 complement (usually 9’s and 1’s complements) of a fractional number can be obtained as:

$$N^{r-1}=r^{n}-r^{-m}-N$$

Where m is the number of digits in the fractional part of ne number N.

Example 7: Find the10’s and 9’s complements of $(21.426)_{10}$

Solution:

10’s complement tümleyen:

$N^{r}=10^{n}-N=100.000-21.456=78.544$.

9’s complement: (n=2, m=3)

r-1 complment=$N^{r-1}=r^{n}-r^{-m}-N$

$N^{r-1}=10^{2}-10^{-3}-21.426=100-0.001-21.426=78.543.$

Example 8 : **Find the 9's complement of the decimal number (725.250).**

Solution:

r = 10, n = 3, m = 3. Threfore 9's complement is:

$10^3 - 10^{-3} - 725.250 = 1000 - 0.001 - 725.250 = 274.749$.

ÖRNEK 9: **Find the 1's complement of the binary number (110.1011).**

Solution:

r = 2, n = 3, m = 4. Therefore 1's is;

$2^3 - 2^{-4} - 110.1011 = 1000 - 0.0001 - 110.1011 = 001.0100$

COMPLEMENT ARİTHMETİC

Substraction using r Complement arithmetic

Diffirence of to positive number with radix r (base r) 'M - N' can be obtained as follows:

a) Add the number M with the r complement of the number N.

b) İf there is a carry at the end, then the carry is ignored. Remaining is the correct result and it is a positive number.

If there is no carry at the end, then the result is the complement of the true result and the result is a negative number. In this case to obtain the true result the r complement of the result is obtained and a minus sign (-) is placed infront of it.

EXAMPLE 1: Using 10’s complement substract the decimal number 3250 from the decimal number 72532.

Solution:

10’s complement of 03250 is:
100000 - 3250 = 96750.
Therefore: 72532 + 96750 = 169282
There is carry 1. Therefore result is a positive number.
After ignoring the carry the result is:
+69282.

Complement Arithmetic

EXAMPLE 2: Using 2's complement substract the subtract the binary number $(1000100)_2$ from the binary number $(1010100)_2$.

Solution:

2's complement of $(1000100)_2$ is;
0111100. The result is:
1010100 + 0111100 = 10010000
Tehre is a carry of 1. İgnoring the carry the true result is the positive number:
+ 0010000 . (72-64=8)

Example 3: Using 10’s complement substract the decimal number 72532 from the decimal number 3250.

Solution:

10’s complement of 72532 is:
100000 - 72532 = 27468.
Therefore: 03250 + 27468 = 030718
There is no carry . Therefore result is a negative number. We need 10’s complement of resulting number 030718:fter ignoring the carry the result is:
100000-030718=69282
Therefore the true result is
(-69282) dir.

Example 4: SİGNED NUMBERS: Add the signed numbers -100 and -27 using 1's complement.

-100=11100100 and 1's complement of -100=10011011
-27=10011011 and 1's complement of -27=11100100

```
   10011011
   11100100
 ___________
(1)01111111
          1      (carry is added to LSB)
 ___________
   10000000
```

10000000 (=11111111 (Complenet after ignoring the carry)=-($2^6+2^5+2^4+2^3+2^2+2^1+2^0$=-(64+32+16+8+4+2+1)=-127

($2^{n-1}=2^{8-1}=2^7=128>127$) (if $2^{n-1}<$ the result then result is not correct due to overflow. (bit yetmezliği)

Example 5: SİGNED NUMBERS: Add the signed decimal numbers +132 and -69 using 1's complement.

Solution:

$(+132)_{10}=(010000100)_2$

$(-69)_{10}=(101000101)_2$

1's complement of $(-69)_{10}=(110111010)_2$

```
      010000100
  +   110111010
  _____________
   (1)000111110
+             1
  _____________
      000111111
```

000111111 Result is positive binary number

$000111111=2^5+2^4+2^3+2^2+2^1+2^0=(63)_{10}$

EXAMPLE PROBLEMS

Q 1 – Convert the following numbers to base 10.

a) $(4310)_5$ b) $(198)_{12}$ c) $(735)_8$ d) $(525)_6$

Solution:

$(4310)_5 = 4 * 5^3 + 3 * 5^2 + 1 * 5^1 =(580)_{10}$

$(198)_{12} = 1 * 12^2 + 9 * 12^1 + 8 * 12^0 = (260)_{10}$

$(735)_8 = 7 * 8^2 + 3 * 8^1 + 5 * 8^0 = (477)_{10}$

$(525)_6 = 5 * 6^2 + 2 * 6^1 + 5 * 6^0 = (197)_{10}$

EXAMPLE PROBLEMS

Soru 2 – Find the 1's complement and 2's complement of the folowing numbers.

a) $(1100110)_2$ b) $(01101101)_2$ c) $(111101)_2$

d) $(23.84)_{10}$ e) $(125.625)_{10}$

Solution:

a) 1's complement: 0011001,

2's complement : 0011010

b) 1's complement : 10010010,

2's complement : 10010001

c) 1's complement : 000010,

2's complement : 000011

EXAMPLE PROBLEMS

d) At first convert decimal number to binary number and then find 1's and 2's complements.

$(23.84)_{10}$ =$(10111)_2$ olur.

1's complement= 01000,

2's complement=01001 olur.

EXAMPLE PROBLEMS

e) At first convert decimal number to binary number and then find 1's and 2's complements.

$(125.625)_{10}=(1111101.101)_2$

1's complement: 0000010.010

2's complement: 0000010.011 (apply the rule that was described in this chapter earlier (keep same until the first 1 and then complement all bits on the right side)).

EXAMPLE PROBLEMS

Q 3: Add and multiply the following numbers without converting to the decimal numbers.

a) $(11001)_2$ ve $(1101)_2$ **b)** $(2AC)_{16}$ ve $(E2)_{16}$

c) $(3A4)_{16}$ ve $(C5)_{16}$

Solution)

$$\begin{array}{rl} 11001 & (=25_{10}) \\ +\ 1101 & (=13_{10}) \\ \hline 100110 & = (38)_{10} \end{array}$$

EXAMPLE PROBLEMS

```
        11001
X        1101
   -----------
        11001
 +    11001
   -----------
      1001011
 +  11001
   -----------
```

$(10010011)_2 = (275)_{10} = (25X11)$

Örnek Soru Çözümleri

b)

$$
\begin{array}{rll}
 & (2AC)_{16} & [\,(684)_{10}] \\
+ & (\;E2)_{16} & [\,(226)_{10}] \quad (684+226=910) \\
\hline
 & (\,38E)_{16} & =(910)_{10}
\end{array}
$$

$$
\begin{array}{rll}
 & (2AC)_{16} & [\,(684)_{10}] \\
X & (\;E2)_{16} & [\,(226)_{10}] \quad (684x226=154584) \\
\hline
 & 558 & \\
+ & 2568 & \\
\hline
 & (25BD8)_{16} & = (154584)_{10}
\end{array}
$$

EXAMPLE PROBLEMS

c)

$$\begin{array}{rl} & (3A4)_{16} \\ + & (\ C5)_{16} \\ \hline & (\ 469)_{16} \end{array} \quad \begin{array}{l} [\ (932)_{10}] \\ [\ (197)_{10}] \quad (932+197=1129) \\ =(1129)_{10} \end{array}$$

$$\begin{array}{rl} & (3A4)_{16} \\ X & (\ C5)_{16} \\ \hline & 1234 \\ + & 2BB0 \\ \hline & (2CD34)_{16} \end{array} \quad \begin{array}{l} [\ (932)_{10}] \\ [\ (197)_{10}] \quad (932x197=183604) \\ \\ \\ = (183604)_{10} \end{array}$$

BINARY CODES

1) BINARY CODED DECIMAL (BCD)

Although most computers uses binary numbers but output is in decimal numbers. Therefore decimal numbers must be coded in binary.

937.25 is coded as:

1001 0011 0111 . 0010 0101

9 3 7 2 5

Each decimal digit is coded in binary.

BCD addition

Example 1:

$146_{10}+259_{10}=405_{10}$

```
  0001 0100 0110        Form BCD
+ 0010 0101 1001
-----------------
  0011 1001 1111
+           0110        correction
-----------------
  0011 1010 0101
+      0110             correction
-----------------
  0100 0000 0101        =405₁₀
```

Example 2:

$52_{10}+199_{10}=251_{10}$

```
   0000 0101 0010        Form BCD
 + 0001 1001 1001
 ----------------
   0001 1110  1011
 +      0110  0110       correction
 ----------------
   0010 0101  0001   =251₁₀
```

Example 3:

$19_{10}+48_{10}=67_{10}$

0001 1001	Form BCD
+ 0100 1000	
0110 0001	(carry 1 from one group to another)
+ 0110	correction
0110 0111	$=67_{10}$

Example 4

Using BCD addition add decimal numbers 17 and 18.

```
  0001 0111
  0001 1000
-----------
  0010 1111
       0110
-----------
  0011 0101   =35(=17+18)
```

Example 5

Using BCD addition add decimal numbers 19 and 18.

```
   0001   1001
   0001   1000
 ______________
   0011   0001
          0110
 ______________
   0011   0111    =37(=19+18)
```

WEIGHTED CODES

In weighted codes each column has a weight . Decimal numbers corresponding to each row Can be obtained by using weights in each clumn. For example 2-4-2-1 code is a wighted code. In this code a row composed of 1101 corresponds to a decimal number of

$1x2+1x4+0x2+1x1=(7)_{10}$ (decimal)

Or

1011 in 6-3-1-1 code corresponds to

1x6+0x3+1x1+1x1=(8)

2) 8-4-2-1 CODE (Wighted code)

8-4-2-1 CODE				
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1

2) 6-3-1-1 CODE (Weighted code)

0	0	0	0	0
1	0	0	0	1
2	0	0	1	1
3	0	1	0	0
4	0	1	0	1
5	0	1	1	1
6	1	0	0	0
7	1	0	0	1
8	1	0	1	1
9	1	1	0	0

3) 4-3-2-1 CODE (Weighted code)

0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	1	0	0
4	1	0	0	0
5	1	0	0	1
6	1	0	1	0
7	1	1	0	0
8	1	1	0	1
9	1	1	1	0

There are many other weighted codes and you may generate your own code.

UN-WEIGHTED CODES (Ağırlıksız kodlar)

4) EXCESS 3 CODE (un-weighted code)

0	0	0	1	1
1	0	1	0	0
2	0	1	0	1
3	0	1	1	0
4	0	1	1	1
5	1	0	0	0
6	1	0	0	1
7	1	0	1	0
8	1	0	1	1
9	1	1	0	0

Excess 3 code is obtained by adding binary 3 (0011) to each raw of 8-4-2-1 code.

5) 2 OUT OF 5 CODE (un-weighted code)

0	0	0	0	1	1
1	0	0	1	0	1
2	0	0	1	1	0
3	0	1	0	0	1
4	0	1	0	1	0
5	0	1	1	0	0
6	1	0	0	0	1
7	1	0	0	1	0
8	1	0	1	0	0
9	1	1	0	0	0

The 2 out of 5 code has the property of exactly 2 out of 5 bits are 1.

6) GRAY CODE (un-weighted code)

0	0	0	0	0
1	0	0	0	1
2	0	0	1	1
3	0	0	1	0
4	0	1	1	0
5	0	1	1	1
6	0	1	0	1
7	0	1	0	0
8	1	1	0	0
9	1	1	0	1

Useful for use in converting measured analog data to digital data (A/D converters), for example when mesuring speed of a motor by using a shaft encoder.

Conversion from 8-4-2-1 code to Gray code:

Let any binary number in 8-4-2-1 code be as:
N=w x y z . Also let Correspondin Gray code be G=abcd. G can be determined as follows:

First bit of the Gray code is the same as first bit, a=w.

Second bit of the Gray code, b=w+x - carry(if exists).

Third bit of the Gray code, c=x+y - carry(if exists).

Fourth bit of the Gray code, d=y+z - carry(if exists).

Examples follow on the next slides.

Exampl 1: Convert number 6 (0110) to Gray code:

First bit of the Gray code=0
Secon bit of the Gray code=0+1= 1
Third bit of the Gray code=1+1= 0 (drop cary 1)=0
Fourt bit of the Gray code=1+0= 1
Therefore 6 (0110) in Gray code= 0101

Exampl 2: Convert number 13 (1101) to Gray code:

First bit of the Gray code=1
Secon bit of the Gray code=1+1= 0 (drop cary 1) = 0
Third bit of the Gray code=1+0= 1
Fourt bit of the Gray code=0+1= 1
Therefore 13 (1101) in Gray code= 1011

SECOND RULE: **Construct a 4 bit Karno map annd place the number 9 to 15 in map as shown. Then assume numbers in Gray code to be G=abcd, a 4 bit number. Then read the ab component of the Gray code, correspondin to a decimal number, from the horizontal axis and the cd coponent from the vertical axis of the map. As an exmple number 7 (0111) in Gray code will be 0100 (ab=01 cd=00).**

cd \ ab	00	01	11	10
00	0	7	8	15
01	1	6	9	14
11	2	5	10	13
10	3	4	11	12

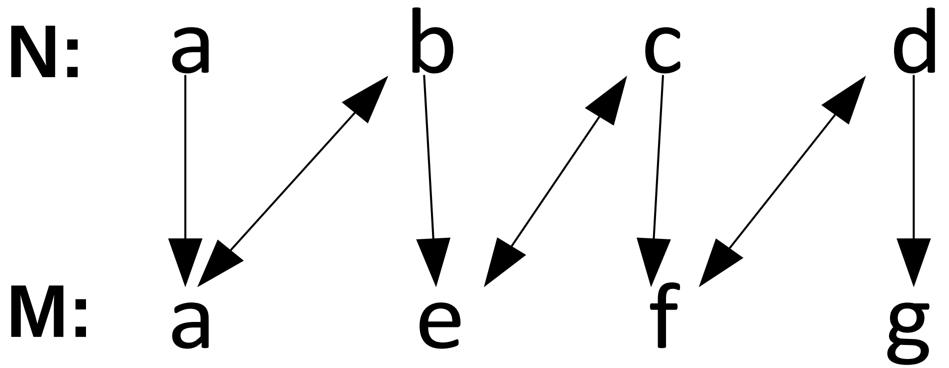
Similarly number 5 (0101) in Gray code will be 0111 (ab=01 and cd=11)

Therefore all four bit numbers (0 to 15) can be written in Gray code as follows:

Decimal number	8-4-2-1 binary number	Numbers in Gray code
0	0000	0000
1	0001	0001
2	0010	0011
3	0011	0010
4	0100	0110
5	0101	0111
6	0110	0101
7	0111	0100
8	1000	1100
9	1001	1101
10	1010	1111
11	1011	1110
12	1100	1010
13	1101	1011
14	1110	1001
15	1111	1000

GRAY CODE TO BINARY (8-4-2-1) CODE CONVERSION

Gray code can be converted to normal binary code (8-4-2-1) using the following rule:
N=abcd (4 bit gray code)
M=aefg (normal binary 8-4-2-1 code)



e =(a+b) - carry (if exists)
f =(e+c) - carry (if exists)
g =(f+d) - carry (if exists)

Example 1 :	**Number 4 :**	**Gray code**	**0 1 1 0**
		8-4-2-1 code	**0 1 0 1**
Example 1 :	**Number 7 :**	**Gray code**	**0 1 0 0**
		8-4-2-1 code	**0 1 1 1**
Example 1 :	**Number 13 :**	**Gray code**	**1 0 1 1**
		8-4-2-1 code	**1 1 0 1**
Example 1 :	**Number 15**	**: Gray code**	**1 0 0 0**
		8-4-2-1 code	**1 1 1 1**

Table 1–2. Binary Codes for Decimal Digits

Decimal Digit	8-4-2-1 Code (BCD)	6-3-1-1 Code	Excess-3 Code	2-out-of-5 Code	Gray Code
0	0000	0000	0011	00011	0000
1	0001	0001	0100	00101	0001
2	0010	0011	0101	00110	0011
3	0011	0100	0110	01001	0010
4	0100	0101	0111	01010	0110
5	0101	0111	1000	01100	0111
6	0110	1000	1001	10001	0101
7	0111	1001	1010	10010	0100
8	1000	1011	1011	10100	1100
9	1001	1100	1100	11000	1101

ASCII KODE

ASCII: American Standard Code for Informatıon Interchange

Character	ASCII Code A_6	A_5	A_4	A_3	A_2	A_1	A_0	Character	ASCII Code A_6	A_5	A_4	A_3	A_2	A_1	A_0
space	0	1	0	0	0	0	0	@	1	0	0	0	0	0	0
!	0	1	0	0	0	0	1	A	1	0	0	0	0	0	1
"	0	1	0	0	0	1	0	B	1	0	0	0	0	1	0
#	0	1	0	0	0	1	1	C	1	0	0	0	0	1	1
$	0	1	0	0	1	0	0	D	1	0	0	0	1	0	0
%	0	1	0	0	1	0	1	E	1	0	0	0	1	0	1
&	0	1	0	0	1	1	0	F	1	0	0	0	1	1	0
'	0	1	0	0	1	1	1	G	1	0	0	0	1	1	1
(	0	1	0	1	0	0	0	H	1	0	0	1	0	0	0
)	0	1	0	1	0	0	1	I	1	0	0	1	0	0	1
*	0	1	0	1	0	1	0	J	1	0	0	1	0	1	0
+	0	1	0	1	0	1	1	K	1	0	0	1	0	1	1
,	0	1	0	1	1	0	0	L	1	0	0	1	1	0	0
–	0	1	0	1	1	0	1	M	1	0	0	1	1	0	1
.	0	1	0	1	1	1	0	N	1	0	0	1	1	1	0
/	0	1	0	1	1	1	1	O	1	0	0	1	1	1	1
0	0	1	1	0	0	0	0	P	1	0	1	0	0	0	0
1	0	1	1	0	0	0	1	Q	1	0	1	0	0	0	1
2	0	1	1	0	0	1	0	R	1	0	1	0	0	1	0
3	0	1	1	0	0	1	1	S	1	0	1	0	0	1	1
4	0	1	1	0	1	0	0	T	1	0	1	0	1	0	0

Table 1-3
ASCII code
(incomplete)

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Örnek Soru Çözümleri

Soru 1 - Aşağıdaki sayıları onluk tabana dönüştürünüz.

a) $(4310)_5$ b) $(198)_{12}$ c) $(735)_8$ d) $(525)_6$

Çözüm:

$(4310)_5 = 4 * 5^3 + 3 * 5^2 + 1 * 5^1 =(580)_{10}$

$(198)_{12} = 1 * 12^2 + 9 * 12^1 + 8 * 12^0 = (260)_{10}$

$(735)_8 = 7 * 8^2 + 3 * 8^1 + 5 * 8^0 = (477)_{10}$

$(525)_6 = 5 * 6^2 + 2 * 6^1 + 5 * 6^0 = (197)_{10}$

Örnek Soru Çözümleri

Soru 2 - Aşağıdaki sayıların 1'e (1's complement) ve 2 'ye (2's complement) tümleyenlerini bulunuz.

a) $(1100110)_2$ **b)** $(01101101)_2$ **c)** $(111101)_2$

d) $(23.84)_{10}$ **e)** $(125.625)_{10}$

Çözüm:

a) 1'e tümleyen: 0011001,
2'ye tümleyen: 0011010

b) 1'e tümleyen: 10010010,
2'ye tümleyen: 10010001

c) 1'e tümleyen: 000010,
2'ye tümleyen: 000011

Örnek Soru Çözümleri

d) Önce sayının binary (ikili) sayıya dönüştürülmesi gerekir. Decimal sayının binary sayıya nasıl dönüştürüleceğini biliyoruz.

$(23.84)_{10} = (10111)_2$ olur.

1’e tümleyen= 01000,

2’ye tümleyen=01001 olur.

Örnek Soru Çözümleri

e) Sayının binary (ikili) dönüşümü:
$(125.625)_{10}=(1111101.101)_2$

1’e tümleyen: 0000010.010

2’ye tümleyen: 0000010.011 (kural kesirli sayı içinde geçerli (soldan itibaren ilk 1’e kadar aynısı sonraki bitlerin tümleyenleri alınır)).

Örnek Soru Çözümleri

Soru 3: **Aşağıdaki sayıları onluk sayılara dönüştürmeden toplayın ve çarpın.**

a) $(11001)_2$ ve $(1101)_2$ b) $(2AC)_{16}$ ve $(E2)_{16}$
c) $(3A4)_{16}$ ve $(C5)_{16}$

Çözüm: a)

$$
\begin{array}{rl}
11001 & (=25_{10}) \\
+\ 1101 & (=13_{10}) \\
\hline
100110 & = (38)_{10}
\end{array}
$$

Örnek Soru Çözümleri

```
        11001
X        1101
      ---------
        11001
   +   11001
   ----------
      1001011
  +  11001
  -----------
```

$(10010011)_2 = (275)_{10} = (25X11)$

Örnek Soru Çözümleri

b)

$$
\begin{array}{rll}
 & (2AC)_{16} & [\,(684)_{10}] \\
+ & (\;E2)_{16} & [\,(226)_{10}] \quad (684+226=910) \\
\hline
 & (\,38E)_{16} & =(910)_{10}
\end{array}
$$

$$
\begin{array}{rll}
 & (2AC)_{16} & [\,(684)_{10}] \\
X & (\;E2)_{16} & [\,(226)_{10}] \quad (684x226=154584) \\
\hline
 & 558 & \\
+ & 2568 & \\
\hline
 & (25BD8)_{16} & = (154584)_{10}
\end{array}
$$

Örnek Soru Çözümleri

c)

$$
\begin{array}{rl}
 & (3A4)_{16} \quad [\,(932)_{10}] \\
+ & (\;C5)_{16} \quad [\,(197)_{10}] \quad (932+197=1129) \\
\hline
 & (\,469)_{16} \quad =(1129)_{10}
\end{array}
$$

$$
\begin{array}{rl}
 & (3A4)_{16} \quad [\,(932)_{10}] \\
X & (\;C5)_{16} \quad [\,(197)_{10}] \quad (932x197=183604) \\
\hline
 & 1234 \\
+ & 2BB0 \\
\hline
 & (2CD34)_{16} \;=\; (183604)_{10}
\end{array}
$$

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